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# ORGANIZED FLIGHT IN BIRDS

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## ABSTRACT

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1 The organized flight of birds is one of the most easily observed, yet challenging to study,  
2 phenomena in biology. Birds that fly in organized groups generally do so in one of two fashions:  
3 Line formations and Cluster formations. The former groups are typically demonstrated by large  
4 birds like waterfowl, where birds fly arranged in single lines, often joined together. The  
5 scientific questions about these groups usually involve potential adaptive functions, such as why  
6 do geese fly in a V? The latter, Cluster formations, are typically made up of large numbers of  
7 smaller birds like pigeons or blackbirds flying in more irregular arrangements that have a  
8 strong three dimensional character. The groups are defined by synchronized and apparently  
9 simultaneous rapid changes in direction. Scientific questions about these groups are usually  
10 concerned with mechanism; how is synchrony achieved? Although field observations about the  
11 phenomenon date to the origins of natural history, experimental studies did not begin until the  
12 1970s. Early experimenters and theoreticians were primarily biologists, but more recently  
13 aeronautical engineers, mathematicians, computer scientists, and currently, physicists have  
14 been attracted to the study of organized flight. Computer modelling of organized flight has  
15 recently generated striking visual representations of organized flight and a number of  
16 hypotheses about the functions and mechanisms of organized flight, but the ability to test these  
17 hypotheses lags behind the capacity to generate them. It is suggested that a multiple-  
18 disciplinary approach to the phenomenon will be necessary to resolve apparently conflicting  
19 current hypotheses.

## 20 Keywords

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21 animat; bird aerodynamics; bird flight; bird flocking; boid; cluster formations; Canada Goose;  
22 *Branta Canadensis*; European Starling; *Sternus vulgaris*; flight formations; flock simulations;  
23 flocking simulations; line formations; V formation

24 The orderly aerial manoeuvres of birds have fascinated and mystified observers since the  
25 beginnings of written natural history 2,000 years ago, when Pliny suggested that geese ‘...travel  
26 in a pointed formation like fast galleys, so cleaving the air more easily than if they drove at it  
27 with a straight front’ (Rackham 1933). Why do geese fly in a V, and how do pigeons all seem to  
28 be able to take off and turn at once? The study of these phenomena offers an encapsulated  
29 model of the development of knowledge of other behaviours, starting with anecdotal  
30 descriptions and speculation, measured observations of increasing precision, formation of  
31 testable hypotheses, and then tests of these hypotheses. In the case of the study of organized  
32 flight in birds, the first phase began at about the beginning of the twentieth century, the second  
33 and third in the 1970s and the fourth in the mid 1980s. The study of bird organized flight also  
34 offers a good demonstration of Kuhn’s (1962) suggestion that science advances in saltatory  
35 fashion, each ‘revolution’ being prompted by a new technique or apparatus that allows old data  
36 to be looked at in a new way.

37 The early investigators of organized flight were, with a few notable exceptions, biologists. In  
38 the 1970s, aeronautical engineers started to be attracted to the phenomenon, followed by  
39 computer scientists in the 1980s, and physicists and mathematicians in the 1990s. These later  
40 investigators have been primarily interested in modelling the behaviour. The fraction of active  
41 investigators with a biological background has steadily decreased over the years. We will try to  
42 demonstrate that as the elegance of models has increased, so has their distance from behaviour  
43 in the field, and that future progress in the area will depend on collaborations between  
44 physicists, mathematicians, computer scientists and biologists rather than specialists working  
45 alone.

## 46 **THE ERA OF ANECDOTE AND SPECULATION**

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47 Several ornithologists of the 1930s made visual field observations that would later be very  
48 provocative to experimentalists and theoreticians. Nichols (1931) noted that in turning and  
49 wheeling pigeon, *Columba livia*, flocks, the position of the birds at the head of a turning flock

50 would be exchanged with birds at the side after the completion of a turn; there did not appear to  
51 be consistent 'leadership' in such flocks. He speculated that this behaviour might be the result of  
52 faster birds in the front of the formation moving ahead of the flock, then turning back to rejoin.  
53 The visual stimulus provided by the turnaround might provide a signal for the rest of the birds  
54 to turn, apparently simultaneously. He suggested that a change in direction was related to a  
55 change of positional leadership.

56 Selous (1931) made a 30-year series of meticulous visual observations on various species of  
57 birds flying in organized flocks, and was convinced that within the limits of unassisted human  
58 vision, there were occasions when birds rose from the ground, or made turns simultaneously.  
59 He concluded that there could be only two possible explanations for such a phenomenon;  
60 disturbance from outside the flock, say the sight of a predator, which would be instantaneously  
61 received by all birds in the flock, and would be reacted to in identical manner, or an undefined  
62 quality he called 'thought transference', or what we might call today 'telepathy'.

63 Selous appeared convinced that there were at least some occasions when groups of birds  
64 would rise from the ground, apparently spontaneously, with no discernible source of outside  
65 disturbance. He also noted in contrast that there were times when a flock on the ground would  
66 be indifferent to the rapid approach of an aerial predator, as when members of a flotilla of  
67 Eurasian Coots, *Fulica atra*, leisurely swam away as a Great Black-backed Gull, *Larus marinus*,  
68 made a low pass over their group. Penrose (1949) made a similar observation when he dove  
69 from above toward a large European Starling, *Sternus vulgaris*, flock in a sailplane.

70 Selous also noted that flocks on the ground would sometimes take to the air in a stepwise  
71 fashion. Individuals or small groups of Black-headed Gulls, *Larus ridibundus*, would take flight  
72 without any discernible effect on neighbours, and then with no obvious temporal relationship to  
73 previous small group departures, the entire remainder of the flock, hundreds of birds, would  
74 take flight simultaneously.

75 'Thought transference' had a different standing in the scientific community in Selous' time  
76 than it does today, and it is not surprising that, for want of a better explanation, a careful

77 observer like Selous might be led to something as heterodox as telepathy to explain an  
78 otherwise inexplicable phenomenon. Rhine (1983) had started reporting the results of  
79 parapsychology experiments using conventional experimental design in 1927, and England,  
80 where Selous made his observations was a centre of interest in 'paranormal' phenomena. Selous  
81 never explored what the nature of thought transference might be.

82 Gerard (1943) was one of the first individuals to try to quantify turning behaviour in a flock.  
83 Whilst pacing a group of approximately 100 unidentified birds in a car being driven at 35 mph  
84 (60 km/h), he observed that the entire flock turned left in a flanking movement, rather than a  
85 column movement, in military parlance. In a flanking movement all individuals turn at once  
86 upon the signal to do so, rather than advancing to a defined point and then turning. He  
87 speculated that no bird advanced more than a body's length beyond any other bird before  
88 turning, by his calculation within 5 ms of any other bird. Assuming a minimum reaction time of  
89 100 ms, he proposed that any coordinating signal must have been acted on with great constancy  
90 by receiving individuals. Gerard's own vision must have been remarkable to be able to make  
91 this observation while driving a car, but his estimate of probable reaction time was very close to  
92 Pomeroy & Heppner's (1977) laboratory study results of startle reaction times in the European  
93 Starling of 70 ms.

94 Much of the early work on flight flocking was devoted to considerations of the biological  
95 utility of flocking, from an ecological or behavioural standpoint, rather than the perspective of  
96 organizing principles or mechanisms. Beer (1958) questioned whether large groupings of birds  
97 had 'any' distinctive utility, and were merely 'haphazard organizations'. Vine (1971), on the  
98 other hand, suggested that a circular grouping provided the best predator avoidance strategy  
99 against visual predators. Emlen (1952) looked at flocks from the ethological perspective of the  
100 times, and suggested that both flocking itself, and the structure of the flock resulted from the  
101 interplay of attractive and repulsive behavioural forces.

102 One of the annoyances that has persisted over the years for those studying flocks is an  
103 etymological one; there has been no consistency in the literature in terms of the definition of

104 'flock' and categories of same. The difficulty is not a trivial one. One author might be describing  
105 the properties of a class of behaviours that is quite different than those studied by a different  
106 investigator, but both will use the same term.

107 For example, Emlen (1952, p. 160) described a flock as 'any aggregation of homogeneous  
108 individuals, regardless of size or density'. This definition immediately presents difficulties,  
109 because there are very common aerial groupings, such as mixed blackbird groups, composed of  
110 different species. Beer's (1958, p. 78) definition of a flock was '...two or more birds which  
111 associate with each other due to innate gregarious tendencies'. This definition breaks down in  
112 the face of more recent flocking studies, like Reynolds' (1987), which suggest that coordinated  
113 flocking may be the product not simply of 'gregariousness', but extremely simple behavioural  
114 rules followed by each bird in the group.

115 Heppner (1974) developed a taxonomy of airborne bird flocks. The primary dichotomy in this  
116 scheme was between 'Flight Aggregations', which are unorganized groups of flying birds  
117 gathered in an area for a common purpose, such as gulls circling about a fishing trawler, and  
118 'Flight Flocks', which were organized groups of flying birds coordinated in one or more aspects  
119 of flight, such as taking off, turning, landing, etc. However, these distinctions seem not to have  
120 been universally adopted in the literature; one regularly sees the term 'aggregation' used to  
121 describe what Heppner would have called a 'flight flock'.

122 Heppner's second order division of 'Flight Flocks' has demonstrated some persistence and  
123 consistency in the literature. He differentiated flight flocks into 'Line Formations' and 'Cluster  
124 Formations' (Fig. 1). Line formations are demonstrated by relatively large birds that fly in  
125 regular lines or queues, such as geese, cormorants, or ducks. Cluster formations have a three  
126 dimensional structure like a sphere, and are typically seen in smaller birds like pigeons,  
127 starlings, and smaller shorebirds. Interestingly, line flying birds like geese may sometimes be  
128 seen in a cluster, but cluster flying birds like starlings are rarely, if ever, seen flying in single  
129 lines.

130 The categories of biological questions that are raised by each of these formations are quite  
131 different. Typically, 'how' questions are raised about cluster flocks. Do the birds really turn all at  
132 once? How can they achieve synchrony in taking off and landing? How do they decide when to  
133 turn, and in what direction? 'Why' questions are more characteristic of line flying birds. What  
134 might be the biological advantage of flying in this configuration? Are there energy savings to be  
135 had? Does the formation shape facilitate communication? A broad question that might apply to  
136 both groups is whether there is a general advantage to flying in groups, as opposed to solitary  
137 flight?

138 A literature search suggests that investigators recognize that the two formation categories  
139 may represent quite different biological issues. Early key papers on line formations tend to be  
140 cited through generations of papers on line formations, but not cluster formation studies, and  
141 vice versa. For this review, we recognize the difference between these lines of investigation, and  
142 will treat them separately.

## 143 **LINE FORMATIONS**

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144 Line flying birds typically fly in staggered, or 'echelon', formations rather than in straight lines  
145 nose-to-tail. If two such formations are joined at an apex at the front of the formation, we have a  
146 V or a J, its asymmetric variant. Franzisket (1951), von Holst (1952) and Hochbaum (1955)  
147 suggested that close formation flight might provide the advantage of a turbulence free zone  
148 behind a bird ahead, but that would seem to apply only if the birds flew immediately behind the  
149 bird in front, like race cars, which they rarely do.

150 Two competing, but not necessarily mutually exclusive, hypotheses have been advanced to  
151 explain the functionality of staggered line formations (most of the papers to be subsequently  
152 cited here refer to V formations and their properties, but Gould & Heppner [1974] found in a  
153 study of 104 Canada Goose, *Branta canadensis*, formations that Vs and Js together were less  
154 common than single staggered lines, or echelons. Additionally, O'Malley & Evans [1982a, b]  
155 found that White Pelicans, *Pelecanus erythrorhynchos*, flying in line formations only flew in Vs

156 10% of the time). Wieselsberger (1914), an aerodynamacist, was the first to suggest an  
157 aerodynamic advantage for line formation flight. He proposed that a V partitioned drag equally  
158 between the two legs of the flight, and that birds flying to the left or right of a bird in front could  
159 recapture energy lost to an 'upwash' generated off the wingtips of the preceding bird. The  
160 alternate, behavioural view suggests that social and perceptual factors have guided adoption of  
161 staggered formations. Hamilton (1967) suggested that flying in staggered lines permitted the  
162 optimum combination of visibility of neighbours, and a clear visual field to the front. Heppner  
163 (1974) suggested that the fixed position of the eyes in the heads of line-flying birds might make  
164 a staggered formation advantageous for keeping the image of an adjacent bird on the visual axis  
165 of a given bird's eyes. However, Heppner et al. (1985) found that the angle between the legs of a  
166 V formation of Canada Geese that would place the image of a leading bird on the visual axis of  
167 the eyes of a following bird ( $128^\circ$ ) was considerably more obtuse than the measured V-angles of  
168 V-formation flying birds in previous studies (Gould & Heppner 1974; Williams et al. 1976;  
169 O'Malley & Evans 1982a). They also noted that although Canada Geese have a limited amount of  
170 binocular vision to the front, despite having eyes located on the sides of their heads, they also  
171 have a 'blind cone' in back of them of  $29^\circ$  on either side of the midline. Thus, a V angle of  $58^\circ$  or  
172 greater would enable every bird in the formation to see every other bird, even those following  
173 behind.

174 Warnke (1984) offered a third hypothesis that, judging by the number of subsequent  
175 citations, seems not to have generated much enthusiasm in the V formation community. He  
176 suggested that the V formation could be explained by the interaction of electrostatic fields  
177 generated by flapping flight. He did not discuss how birds would be able to detect such fields,  
178 nor did he explore the advantage that might accrue to a bird by basing its proximity to a  
179 neighbour on the basis of these fields. There was much interest in the biological effects of  
180 electromagnetic fields in the 1970s and 1980s; Heppner & Haffner (1974) suggested that  
181 coordinated cluster flocks might be explained by signals sent by a leader to all birds in a flock by  
182 means of a hypothetical radiated electromagnetic field. Interestingly, Hill (1972) described a



183 device he patented that was a wing-levelling autopilot for model aircraft that operated by the  
184 differential in electrostatic fields between wingtips of a moving model airplane, so Warnke's  
185 idea was not totally implausible.

186 There have been more papers addressing the aerodynamic hypothesis of staggered flight than  
187 other hypotheses, and a bit of aerodynamic theory here will make the subsequent references  
188 more intelligible. For a bird to fly by use of a wing requires a 'relative wind'; a passage of air  
189 over the wing. If the front of the wing is tilted up slightly relative to the wind, the relative wind  
190 is deflected downward. The result is a positive force on the underside of the wing, 'Newtonian  
191 lift' (Fig. 2). Additionally, airplane wings are typically constructed so that the top of the wing is  
192 curved and air moves faster over the top of the wing, creating a negative pressure on the top of  
193 the wing; 'Bernoulli lift'. In still air, we must generate the relative wind by moving the aircraft  
194 forward. To do so, we must have a force called 'thrust', generated, for example, by a propellor. In  
195 general, the faster the aircraft goes, the more lift is generated by the wing. Unfortunately, as the  
196 aircraft accelerates, 'drag' is produced, at least in part by friction between the air and the  
197 surface. One type of drag, 'induced drag', is especially germane to bird flight. Lift is, partly,  
198 created by the angle with which the wing meets the air (referred to also as the angle of attack).  
199 Up to a limit, the steeper the angle, the greater the lift—but also, the greater is the induced drag,  
200 which is produced as a by-product of lift. Compared to airplanes, birds are typically low-speed  
201 aircraft whose wings produce a lot of induced drag.

202 The inner part of a bird's wing provides most of the lift, the outer part, by a kind of rowing  
203 action, provides the thrust. As air streams over a wing generating lift, it tends to form vortices,  
204 which typically stream off the wing as 'tip vortices', essentially horizontal tornadoes. These tip  
205 vortices have a rising and falling component, and in an airplane, may carry sufficient energy to  
206 upset a smaller aircraft following a larger one in for a landing. It is this energy, which essentially  
207 represents a cost of flight using wings, that the aerodynamic hypothesis of V formation suggests  
208 might be partially recaptured by a following bird whose own wingtip was located in the upward  
209 rising part of the tip vortex, or upwash, streaming off the wing of the preceding bird (Fig. 3). The

210 diameter of the vortex increases with distance from the producing bird's wingtip, and tends to  
211 dissipate with increasing distance. The placement of a following bird's wingtip in relationship to  
212 the vortex from a preceding bird's wingtip should, in theory, affect how much energy is  
213 recaptured by the following bird. To recapture tip vortex energy from a preceding bird, a  
214 following bird would have to be positioned to the left or right of a preceding bird, suggesting  
215 that a V (or at least a staggered, or echelon) formation would be advantageous for birds flying in  
216 a group.

217 Lissaman & Schollenberger (1970) produced the first quantitative suggestion, based on  
218 aerodynamic theory, of exactly how much energy might be saved by a group of birds flying in a  
219 V formation. They proposed that a group of 25 (unspecified species) birds flying in a V would  
220 have 71 percent more range than a single bird. Their optimum V angle appeared to be about  
221 120° between the legs of the V. For later investigators, this paper was both stimulating and  
222 frustrating because they did not present the calculations and formulae used to arrive at their  
223 conclusions, ignored the quantitative effects of flapping rather than fixed wing flight, and did not  
224 apparently consider the difference between air flowing over a smooth metal surface and a  
225 feathered wing nor the aerodynamic scaling effects of small birds flying at low speeds compared  
226 to aircraft. Nonetheless, this paper provided a 'target' for experimental and quantitative  
227 observational work.

228 Haffner (1977) flew Budgerigars, *Melopsittacus undulatus*, in a wind tunnel and used a smoke  
229 stream to visualize the airflow over the bird's wing. He concluded that flapping wing flight is  
230 aerodynamically complex, and that calculations of energy saving for the V formation using fixed  
231 wing models were oversimplified, and probably overgenerous. Using Cone's (1968) theoretical  
232 studies on flapping wing flight and his own experimental work, he concluded that potential  
233 energy saving of V formation flight compared to solitary flight was a much smaller maximum of  
234 22%.

235 Willis et al. (2007) examined the theoretical energy savings in formation flight with respect to  
236 basic positioning and wing beat phase relationships between a preceding bird and a following

237 bird. Nachtigall (1970) found a phase synchrony in a field study of wing beats in Canada Goose  
238 formations, but Gould (1972), in a similar study, failed to do so. Willis et al.'s (2007) study is  
239 preliminary as they do not consider the optimal formation shape or detailed flapping kinematics  
240 or wing shapes. Nonetheless, their results suggest that optimal flapping phase synchrony  
241 accounts for up to 20% of induced flight power savings, but that precision phase locking is not  
242 required for energy savings to occur. They also observed that ideally, the following bird would  
243 not be vertically elevated above or below the lead bird's wake if flapping started in phase. If  
244 flapping is not in phase, however, it may be advantageous to take on a vertical displacement  
245 relative to the preceding bird to most effectively capture its strongest upwash regions. They  
246 suggest that vertical displacements in nature probably do not happen for aerodynamic benefit,  
247 as for that to occur precision flight dynamics and sensing would be required.

248 Determination of the distance between birds, and the angle of the legs of the V would be  
249 necessary to test V formation hypotheses. Gould & Heppner (1974) performed the first field  
250 measurement of both parameters in Canada Geese using projective geometry and still  
251 photography. They reported a mean angle between the legs of the V  $\pm$ SD of  $34\pm 6^\circ$ ,  $N=5$ , with a  
252 mean distance between bird bodies  $\pm$ SD of  $4.1\pm 0.8$  m,  $N=3$  and a mean flock size  $\pm$ SD of  $18\pm 12$   
253 birds,  $N=5$ . Two years later, Williams et al. (1976) examined V angles in Canada Goose  
254 formations using a radar technique. They found a range of  $38\text{--}124^\circ$  in the feeding flights they  
255 recorded. Further, they noted that the angle in a single formation varied from  $5\text{--}40^\circ$  between  
256 successive sweeps of the radar beam (duration of sweep not reported). Both groups of authors  
257 used their respective photographic and radar techniques on the same flocks of birds in 1975,  
258 and found no significant difference between the two.

259 Higdon & Corrsin (1978) refined Lissaman & Schollenberger's (1970) hypothesis by  
260 considering the effects of flying in three-dimensional fashion, i.e. in a cluster, like starlings. As  
261 one might suspect, the physics is considerably more complex, but they suggested that it was  
262 aerodynamically disadvantageous to fly directly behind another bird, and that a tall, narrow  
263 cluster flock (such as is often seen in mixed blackbird flocks) is aerodynamically

264 disadvantageous compared to solitary flight. May (1979) also re-examined Lissaman &  
265 Schollenberger's (1970) suggestions, and concluded that the aerodynamic advantage of line  
266 flight in large birds was 'slight', perhaps as little as 10% compared to solitary flight.

267 Badgerow & Hainsworth (1981) re-examined Gould & Heppner's (1974) data on distances  
268 between Canada Geese to obtain 'wingtip spacing', a variable they felt was more appropriate  
269 than 'distance between body centres' in testing the aerodynamic hypothesis of V formation  
270 flight. When they did this, they found a number of birds had wingtips that overlapped the  
271 position of the wingtips of a bird ahead, a problematic situation for producing an energy  
272 advantage in Lissaman & Schollenberger's (1970) hypothesis. In contrast to Lissaman &  
273 Schollenberger's (1970) predicted maximum range increase of 71% for V formation flight,  
274 Badgerow & Hainsworth's (1981) revision predicted a maximum increase of 51%, with a range  
275 increase of 2–23% for the birds in a selected Gould & Heppner (1974) flock. Hainsworth (1987)  
276 later provided an excellent description of the modified projective geometry technique he and  
277 Badgerow used in the study above for examination of goose flocks, and applied it to his own  
278 photographs of Canada Goose flocks. He noted that birds frequently shifted positions laterally  
279 relative to the bird ahead, although the basic energy saving model of Lissaman & Schollenberger  
280 (1970) predicted that there was an optimum position for energy saving. Using their model, he  
281 concluded that the goose flocks he filmed were only enjoying a 36% energy advantage over  
282 solitary flight, about half of the Lissaman & Schollenberger (1970) model. He cautioned against  
283 a simplistic engineering model for explaining in toto a behaviour that might be highly variable,  
284 depending on circumstance.

285 O'Malley & Evans (1982a, b) broadened the examination of line formation flight by studying  
286 line flight in White Pelicans, *Pelecanus erythrorhynchos*. They used a variant of Gould &  
287 Heppner's (1974) projective geometry technique to measure angles of Vs and Js, and distance  
288 between birds, with much larger sample sizes (45 flocks) than in the Gould & Heppner (1974)  
289 study. The angles ranged from 24–122°, with a mean  $\pm$ SD of  $67\pm 8^\circ$ ,  $N=12$ , for V formations, and  
290  $70\pm 5^\circ$ ,  $N=33$ , for J formations. As in the earlier goose formation measurements, there was wide

291 variation in the measured angles, and the means were well below Lissaman & Schollenberger's  
292 (1970) predicted optimum angle of 120° for maximum aerodynamic advantage. Again, as in the  
293 goose studies, V formations were less common than single line formations.

294 Hummel (1983), an aerodynamacist, further refined the theoretical aspects of formation flight  
295 by considering wing shape, homogeneous vs. non-homogeneous spacing, size of bird, flight  
296 speed, and straight vs. curved lines. He concluded that, under optimum conditions of the above,  
297 energy savings for formation flight were possible due to aerodynamic considerations, but the  
298 wide variance seen in the arrangements of flocks in the field suggested that aerodynamics might  
299 not be the only factor in formation flight.

300 Badgerow (1988) took a fresh look at the aerodynamic and visual hypotheses, and tried to  
301 organize the scant real field data in such a way that they could be subject to test. He suggested  
302 that if aerodynamic advantage was the primary driver of line flight, there should be a certain  
303 geometric relationship between birds in a formation, but if visual considerations were  
304 paramount, there should be a different configuration. Unfortunately, the variation in data  
305 between flocks was sufficiently large to prohibit a clear distinction between the hypotheses,  
306 although Badgerow felt that there was a non-trivial (about 10%) energetic advantage of  
307 formation flight over solo flight.

308 Cutts & Speakman (1994) also found wide variation in placement of individuals in their study  
309 of formation flight of Pink-footed Geese, *Anser brachyrhynchus*. They photographed 54 skeins  
310 from directly beneath, simplifying the extraction of distances and angles. They found that large  
311 numbers of birds flew outboard of the position predicted by theory to maximize aerodynamic  
312 savings, resulting in a postulated mean energy saving of 14%. Further, after a discussion of  
313 optimum flight speed for optimum range, they suggested that if the birds in their sample flew at  
314 a speed that would maximize their range, the savings would drop to 2% of that predicted by  
315 Lissaman & Schollenberger (1970). Speakman & Banks (1998) later used the same technique to  
316 photograph 25 formations of Greylag Geese, *Anser anser*. They found a great deal of variation in  
317 positioning and that only 17% of birds flew in the predicted optimum position for aerodynamic

318 savings. They suggested, using the same assumptions as the Cutts & Speakman (1994) paper,  
319 that the mean saving in induced power was 27%, and the reduction in total flight costs was  
320 5–9% of the whole. Hainsworth (1988) also found in film studies of Brown Pelicans, *Pelicanus*  
321 *occidentális*, that there was wide variation in wingtip spacing, and that there was no evidence  
322 that the birds spaced to optimize possible aerodynamic effects.

323 Shortly after the turn of the new century, several papers appeared with a decidedly more  
324 mathematical bent than had been seen previously, from investigators with backgrounds in the  
325 control of multiple autonomous unmanned aerial vehicles, like the Predator, and Global Hawk.  
326 Seiler et al. (2002) noted the wide variation in distances and angles reported in bird formations  
327 in previous field studies, and in a rather puzzling table suggested that the average number of  
328 birds in a V formation appeared to be small, typically under 10 birds. Other studies (Gould &  
329 Heppner 1974; Hainsworth 1987) had reported mean V formation sizes closer to 20 birds.  
330 Seiler et al. (2002, p. 122) noted that, on theoretical grounds, maintenance of a specific spacing  
331 and angular relationship between a ‘leader’, and following autonomous robotic vehicles is a  
332 daunting task, and that errors in spacing rapidly multiply with each subsequent vehicle, so  
333 much so that ‘—flying in close formation is *not possible* (italics added) with information only  
334 about the predecessors’. In other words, if a vehicle attempts to maintain position in the  
335 formation only by maintaining position with its immediate predecessor in line, the formation  
336 itself will quickly break down. However, they proposed two potential resolutions:

- 337 1. the formations should be very small, and/or
- 338 2. leader positional information should be simultaneously communicated to all  
339 members of the formation; in other words, a trailing bird should maintain its position  
340 with respect to the leader, rather than its immediate predecessors.

341 The same team expanded this idea, and explored the concept of ‘string instability’, the  
342 phenomenon where the trailing vehicle in a line has such difficulty tracking predecessors that it  
343 oscillates in position to such a degree that it eventually cannot stay with the formation (Seiler  
344 et al. 2003). In particular, they explored the difficulties of maintaining lateral positioning in a

345 line formation. They proposed that the difficulty in maintenance of position increases markedly  
346 with position back from the leader; the 'positional error' (assuming the birds were 'trying' to  
347 maintain an optimum position for either aerodynamic or visual reasons) of the number four  
348 bird in relation to the lead bird would be twice as much as that of the number two bird. Seiler  
349 et al. (2003, p. 279) concluded by suggesting '—that (avian) formation flight is *inherently* (italics  
350 added) difficult'. A glance overhead at a winter waterfowl assembly area displaying a panorama  
351 of dozens of birds flying in each of hundreds of separate line flocks suggests a variant of the  
352 catch phrase of the late Spanish ventriloquist, Señor Wences, 'Difficult for me; easy for you'.  
353 Seiler et al. (2003) suggested that their hypothesis could be tested by examining whether birds  
354 further back in the formation have a greater variation in wingtip spacing than those closer to  
355 the leader. This hypothesis, of course, rests on the prior hypothesis that there is an optimum  
356 spacing that the birds are attempting to maintain.

357 Weimerskirch et al. (2001) have provided the best (and to date) most realistic attempt to  
358 resolve in the field whether there is an energy advantage to line formation flight. They trained a  
359 flock of eight Great White Pelicans, *Pelecanus onocrotalus*, to fly in formation behind a  
360 motorboat. Energy consumption during flight was not recorded directly, but inferred from heart  
361 rate data. They measured heart rate from selected individuals in the flock, and from a solitary  
362 bird flying under the same conditions. Heart rates of the birds in formation were 11–15% lower  
363 than that of the solitary bird. From this, they concluded that they had provided empirical  
364 evidence of an aerodynamic advantage to formation flight, in about the same fractional  
365 proportion as the heart rate difference.

366 An alternate interpretation of the data is possible, especially given the relatively scant  
367 proposed saving compared to most aerodynamic theory-based predictions. Pelicans are highly  
368 social animals and the experience of flying solo might have been stressful compared to normal  
369 social flight. Späni et al. (2003) found that laboratory mice housed individually had a heart rate  
370 4% higher than that of mice housed in pairs. So the effect seen may have been due, at least in  
371 part, to social stress rather than aerodynamic advantage.

## 372 Modelling, Simulations and Application

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373 The development of very powerful, relatively inexpensive computers in the late 1990s  
374 permitted a more sophisticated mathematical analysis of V formations. The first to report a  
375 model producing V formations was Flake (2000, pp. 270–275), who extended Reynolds' (1987)  
376 model (to be discussed later) with an additional rule; each artificial bird, or 'animat' (Wilson  
377 1985; Watts 1998), 'attempted' to move laterally away from any animat that blocked its view to  
378 the front, and with that achieved V-formation flocks.

379 Assuming that there is, in fact, a reduction in collective aerodynamic drag experienced by  
380 members of a flock in a V, Dimock & Selig (2003) went a step further and developed a computer  
381 simulation that actually modelled the induced drag. They extended Reynolds' (1987) model to  
382 'detect' potential drag reductions by adding a rule by which each animat acted to reduce the  
383 drag, and observed how the animats self-organized themselves. There was an evolutionary  
384 component to this study—they used genetic algorithms to evolve the model's parameters and as  
385 each animat acted so as to reduce its own drag, the collective result was that the drag reduction  
386 of the flock as a whole was maximized. Limiting the utility of the model, their induced drag  
387 calculations were based on a rigid wing just as Lissaman & Schollenberger's (1970). In relatively  
388 short simulations, their model correctly penalized collisions, and ultimately produced  
389 rigid/stable flocks of perfect Vs. Using the same evolutionary theme, Andersson & Wallander  
390 (2004) suggested that kin selection might explain why there appeared to be so much variation  
391 in V formation structure. Most aerodynamic advantage studies propose that the lead position is  
392 to some degree less advantageous than following positions, but Andersson & Wallander (2004)  
393 suggested that if the flock is composed of kin, the leader might enjoy a gain in inclusive fitness,  
394 even if at a personal energetic disadvantage. A casual glance at feeding or migrating flocks  
395 suggests considerable shifting of position, and 'leadership' changes within the flock, but it would  
396 be useful if there were a quantitative study indicating whether all or most birds assume the  
397 'leader' position during a flight.



398 Nathan & Barbosa (2008) developed a comprehensive computer model that produced V  
399 formations. Their model evolved from a series of simulations that yielded cluster flocks  
400 (discussed below). The animats in their model followed simple rules; each bird attempted to  
401 seek the proximity of the nearest bird (while avoiding collision), each bird attempted to find a  
402 position that offered an unobstructed longitudinal view (if the first rule was not applicable), and  
403 each bird attempted to position itself in the upwash of a leading bird. Using these rules, they  
404 were able to produce Vs, Js and echelons; as well as inverted Vs which are rarely seen in nature.  
405 The model was limited in its ability to handle flock turning movements as it assumed a constant  
406 heading and the rules produced only lateral displacements. An attractive feature of the model  
407 was, nonetheless, that it offered the opportunity to test the relative importance of aerodynamic,  
408 or communication hypotheses, by changing the values of parameters.

#### 409 So, Why do Birds Fly in a V Formation?

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410 After over 30 years of active interest in the field, we may be reasonably certain of the following  
411 things;

- 412 1. Many large birds (but not all) fly in line formations; small birds almost never.
- 413 2. The V and J formations are the most striking and eye catching line formations for  
414 humans to observe, but they are not the most common for birds to fly in; the echelon  
415 has that distinction.
- 416 3. There is wide variation, from flock to flock and species to species, in positioning and  
417 distances of individual birds in a line. Aerodynamic theory predicts, however, that  
418 there is an optimum position and distance between birds if aerodynamic advantage is  
419 to be maximized, both for individuals and flocks.
- 420 4. The lines are wavy as often as they are straight.

421 One of us (FH) once asked a WWII B-17 pilot why bombers flew in a V. His reply was, 'To keep  
422 a clear field of fire for the guns to the front, and to keep an eye on the leader, who does the  
423 navigation.' Birds clearly need not worry about the former, but in fact the 'leader' is

424 determining the direction the flock is to take, it would be an advantage to keep it in sight, an  
425 advantage in a large flock accruing to a curved or irregular line.

426 Why not fly directly to the side of the leader, or directly in back? If a bird flew to the  
427 immediate left or right of another bird, a gust of wind or a startled response from the neighbour  
428 might precipitate a collision. Similarly, if the bird ahead were to suddenly slow down for any  
429 reason, a rear-end collision might be possible. On an uncrowded motorway, drivers rarely  
430 prefer to drive for long distances alongside a car in an adjacent lane, or tuck in close behind a  
431 leading car if there is an opportunity to pass, possibly for similar anti-collision reasons. If the  
432 object of the staggered line formation is primarily to avoid collision while keeping a leader in  
433 sight, one would expect to see wide variation in spacing and alignment, simply because there is  
434 no particular advantage to one spatial relationship rather than another. Similarly, one would  
435 expect to see undulations in the line. As the body of a neighbour momentarily blocked the view  
436 of the leader, perhaps due to a wind gust, an individual bird could simply speed up a bit or drop  
437 back to regain sight of the leader, thus precipitating a wave.

438 But what of the potential aerodynamic advantage of V flight? Aerodynamic theory suggests  
439 that one exists, under certain conditions. One must ask about its relative importance and need,  
440 however, as it is noted that most of the field studies of line formations have not been made on  
441 migration flights, where energy savings, even small ones, might well be of importance, but on  
442 short feeding flights of 10–20 km, where the energy expended in flight represents a small  
443 fraction of the birds' daily energy budget, and that whereas staggered lines are common, Vs and  
444 Js are much less so. We simply do not know what kinds of formations large birds use on their  
445 long migratory flights, which are often over water. Additionally, there may be an energetic cost  
446 to flying in close formation. The stress level in flying in very close proximity to other birds, with  
447 consequent collision risk, might (on migration flights) raise metabolic levels enough to partially  
448 negate any aerodynamic energy advantage of close formation flight.

449 The 'crucial' experiment, to determine if, in formation flight, there is a worthwhile energy  
450 advantage to be gained for aerodynamic reasons, might be to train a group of imprinted line-

451 formation birds like geese to fly in a wind tunnel, and then use modern airflow visualization  
452 techniques to empirically determine what the upwash properties of birds flying in formation  
453 really are (Pennycuick et al. 1997; Rayner 1995).

## 454 **CLUSTER FLOCKS**

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455 There is an extensive literature discussing the biological value of flocking in general (Krebs &  
456 Barnard 1980), but very few papers have appeared with specific reference to the highly  
457 organized turning and wheeling ('cluster') flocks of some small birds. The most commonly  
458 offered hypothesis is that the closely spaced cluster flocks offer protection against aerial  
459 predators like hawks, presumably by increasing the risk of collision to the predator (Tinbergen  
460 1953). Examples have been reported where flocks of starlings and shorebirds bunch up tightly  
461 when attacked by a hawk (Major & Dill 1978). This hypothesis appears reasonable, but leaves a  
462 commonly seen behaviour in some cluster flying species to be explained. At sunset, or just  
463 before, large flocks of European Starlings will form over a roost from smaller foraging flocks  
464 that have dispersed during the day from that roost. These flocks will engage in some of the most  
465 spectacular group movements seen in flocking birds for periods of 30–45 min before settling  
466 into the roost for the night. Two questions immediately present themselves: 1) Do not these  
467 movements 'waste' energy in species for which energy is important (Hamilton et al. 1967)? and  
468 2) by occurring every night in the same location, and being highly visible from up to a km away,  
469 do they not almost invite predator attack? A loitering predator would have an excellent  
470 opportunity to pick off a straggler (we have seen many pre-roosting turning and wheeling flocks  
471 that generate stragglers as the flock splits and rejoins). Why do these flocks not land  
472 immediately in the roost after returning from foraging, and why are there often 10–15 min of  
473 coordinated turning and wheeling before a flock descends to a feeding area, both expending  
474 energy, and facilitating predation?

475 Wynne-Edwards (1962) proposed instead that these movements represented 'epideictic'  
476 displays that might enable individual flock members to assess the population numbers and

477 density of the flock as a whole; information that might be used in regulating breeding behaviour.  
478 This suggestion was part of a larger concept that is called today 'naïve group selection'. This  
479 hypothesis received little support at the time (Crook 1965), but has enjoyed a recent re-  
480 examination (Wilson & Wilson 2007) that may be useful in considering how organized flight  
481 evolved. Another hypothesis was provided by Major & Dill (1978) who suggested that these  
482 turning and wheeling movements were 'protean' (Driver & Humphries 1970); irregular  
483 movements designed to confuse potential predators. A number of recent studies (Biro et al.  
484 2006; Codling et al. 2007; Dell'Arciccia et al. 2008) have suggested that flying in a group  
485 improves homing performance in pigeons, but it is not clear that the structure of the flock has  
486 anything to do with this improvement.

487 There are several questions that are usually asked when considering the mechanism, rather  
488 than the function of cluster flocks:

- 489 1. Do the flock members truly turn simultaneously during a turning movement, or is  
490 there a wave of movement starting at a centre somewhere and passing through the  
491 flock?
- 492 2. Is there a leader in the flock who communicates an action intention in some fashion to  
493 other members of the flock, or is there some emergent property of flocking itself that  
494 produces coordinated movement?
- 495 3. What mechanism governs the departure of flocks from the roost, ground, or perch, in  
496 which sometimes the whole flock departs, and at other times, subgroups will depart  
497 before the main group?

498 Simultaneity (or not) of individuals making a turn has relevance to the related question of  
499 leadership. A wave of turning in the flock would suggest (but does not necessarily provide  
500 evidence for) a relatively simple model where a leader turns, followed by a turn by neighbours  
501 after a suitable reaction time (which was established by Pomeroy & Heppner [1977] to be under  
502 100 ms in laboratory studies of starlings), then a wave passing through the flock as birds  
503 respond to a turn by birds distant from the leader, but who ultimately have responded to a turn

504 initiated by the leader. Vision would be the most parsimonious medium for information  
505 transmittal in such a model. If birds turn simultaneously instead (within the limits of the  
506 recording instrumentation), the question becomes more interesting; either a putative leader has  
507 to communicate a message instantaneously to all members of the flock, seemingly ruling out  
508 sound and vision in large flocks (because the bodies of nearby neighbours would block the view  
509 of more distant birds), or it would be necessary to propose an organizing principle that could  
510 produce synchronized turns without leadership. Such a model only became available in the  
511 1980s.

512 Davis (1980) filmed turning flocks of Dunlin, *Calidris alpina*, with a slow-motion cine camera  
513 (72 frames/s). Dunlin are differentially coloured on their dorsal and ventral surfaces, and Davis  
514 observed that some individuals in flocks of approximately 40 birds all appeared to turn within  
515 120 ms, giving the appearance of a 'flash'. Potts (1984) using a similar technique with Dunlin,  
516 noted some examples of waves of turning that propagated from neighbour to neighbour within  
517 14 ms, considerably faster than the measured reaction times in birds. He proposed a 'chorus-  
518 line' hypothesis to account for rapid turns, in which one bird or a small group could initiate the  
519 movement, which would then be followed by neighbours who responded to their immediate  
520 neighbours and whose speed of response would depend on their own reaction times, but more  
521 distant birds would be able to estimate and anticipate the passage of the wave, as in the  
522 'Mexican wave' in stadiums (Farkas et al. 2002). However, Heppner (1997) suggested the  
523 possibility that a perceived wave of turning in differentially turning birds might be an artefact of  
524 observer position relative to the near and far borders of the flock, and that individuals in a flock  
525 apparently turning in wave fashion might in fact be turning nearly simultaneously.

526 Early on it was realized that to approach these questions, some idea of the geometric  
527 relationship between birds in a cluster flock would be needed, and that meant the development  
528 of three-dimensional (3D) analysis techniques. These techniques are well developed for  
529 laboratory studies of fish schools (Partridge et al. 1980), but are much more challenging for  
530 birds in the field.

531 Major & Dill (1978) obtained the first 3D measurements of distances between birds in free-  
532 flying flocks of Dunlin, *Caladris alpina*, and European Starlings by using a stereoscopic  
533 photographic technique that utilized two 35 mm film cameras whose optic axes were parallel,  
534 and which were firmly fixed on an aluminium bar 5.5 m long. They were particularly interested  
535 in nearest-neighbour distances, and the angles between neighbours, as these would provide an  
536 index of condensation of the flocks. They reported that the nearest neighbour to a reference  
537 bird was typically behind and below a reference bird, a pattern often seen in fish shoals.

538 Pomeroy (1983) and Pomeroy & Heppner (1992) used an orthogonal 3D photographic  
539 technique to obtain sequence pictures of semi-domestic Rock Pigeons, *Columba livia*, turning in  
540 flocks of 8–11 birds. Using this technique, they were able to plot the flight paths of individual  
541 birds, as well as nearest neighbour distances. They reported that the flight paths of individual  
542 birds crossed over each other, such that in a 90° turn, a bird that had been in the lead would be  
543 to the right or left of the flock, and after a 180° turn, would be in the rear of the flock. They  
544 suggested that an individual bird would find it difficult to ‘lead’ a flock by positioning itself at  
545 the head of the flock.

546 Ballerini et al. (2008a, b) and Cavagna et al. (2008a, b) have developed a powerful tool for the  
547 analysis of cluster flocks by essentially solving the ‘correspondence problem’ that has bedevilled  
548 photographic 3D analysis techniques. Most such methods involve taking a pair of pictures from  
549 slightly different viewpoints, and noting the displacement of the image of a single bird in one  
550 view from the other member of the pair. With large numbers of birds of identical appearance,  
551 how do you match the image of the same bird in the two views? By using a novel statistical  
552 method, they were able to determine the positional relationships of over 1,000 birds in a  
553 European Starling flock flying over Rome. Using data obtained by this technique, Ballerini et al.  
554 (2008a) suggest that the significant factor in determining interaction between birds in cluster  
555 flocks is not the distance between birds (‘metric’ distance), but the number of birds between  
556 any two birds (‘topological’ distance, in their terminology).

## 557 Modelling, Simulations and Application

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558 Davis (1980), after reviewing the deficiencies of a leadership model for cluster flock turning and  
559 wheeling movements, suggested the possibility that a 'self-generated synchronous activity'  
560 might provide a model for coordinated movements. Within a decade, the development of  
561 accessible and powerful computers and programming languages produced such models.  
562 Working independently, Okubo (1986), Reynolds (1987) and Heppner & Grenander (1990) each  
563 developed flock flight models based on the concept that each bird in a flock followed simple  
564 behavioural rules in relation to its neighbours, and that the interaction of these rules produced  
565 the emergent property of a coordinated flock. Moreover, Okubo (1986) and Reynolds (1987)  
566 suggested that the same concept could be employed to model schools and herds, which lead to  
567 numerous studies in all three fields based on similar models, as if to find a universal theory. To  
568 some extent the conceptual ancestor of all models was John Conway's 'Game of Life', (Gardner  
569 1970), one of the first cellular automata that demonstrated how complex global behaviours can  
570 arise as a product of self-organization by simple components following simple local rules.

571 To be specific, Reynolds (1987) proposed that for the purpose of computer animation a flock  
572 could be modelled as a group of animats (or 'boids', using his terminology) that followed three  
573 simple rules, which might behaviourally be rephrased as 'drives'. These drives caused the  
574 animats to attempt to avoid collisions with nearby neighbours ('separation' or 'repulsion'),  
575 match velocity with nearby neighbours ('velocity matching'), and stay close to nearby  
576 neighbours ('cohesion' or 'attraction'). The term 'nearby' was used to describe the animat's  
577 localized perception of the universe. In all incarnations of the model, Reynolds (1987, 1999,  
578 2004) used drive dependent perception volumes (nearby neighbours were all animats within a  
579 sphere of a predefined diameter centred at the currently observed animat's origin) with a  
580 biologically realistic perception model (limitations of visual perception were accounted for; a  
581 'blind cone' [the three-dimensional equivalent of a 'blind spot'] was subtracted from the  
582 perceptual sphere at the back of the observed animat). At the time of proposal the approach  
583 represented a giant step forward compared to the traditional techniques used in computer

584 animation for motion pictures. The first animation created with the model was 1987's '(Stanley  
585 and Stella in) Breaking the Ice', followed by a feature film debut in Tim Burton's 1992 film  
586 'Batman Returns' with computer generated bat swarms and 'armies' of penguins marching  
587 through the streets of Gotham City. The current state of the art in computer animation for  
588 motion pictures has evolved even further (Massive 2008); these advanced models, however, due  
589 to the obvious financial consequences, remain proprietary.

590 Heppner & Grenander's (1990) distinguishing features were the approach used to model  
591 perception and the animats' drives. In their case the same perception volume (a sphere of a  
592 predefined diameter centred at the currently observed animat's origin) was used for all drives  
593 and limitations of visual perception were not accounted for. The animats attempted to stay in  
594 the roosting area ('homing'), attempted to fly with a predefined flight speed ('velocity  
595 regulation'), and attempted to move apart if too close, or closer if not too distant ('interaction').  
596 An additional feature was 'random impact', which was intended to simulate the random  
597 distractions that are present in a natural environment (wind gusts, distractions from moving  
598 objects on the ground, etc.). Heppner & Grenander (1990) implemented the latter by using a  
599 Poisson stochastic process and admitted that without its inclusion they were unable to produce  
600 a flock-like behaviour.

601 In the mid 1980s and early 1990s, computer processing power was limited and real time  
602 simulations of large flocks consisting of more than a few dozen birds were infeasible. The first  
603 step to simulations that would allow observing an animated output while running the  
604 simulation and interactively changing the model's parameters was performed by Lorek & White  
605 (1993), who, just a few years after Reynolds' (1987) paper was published, used a Meiko  
606 Transputer System with up to 50 processors to run flight flock simulations, consisting of merely  
607 100 birds at slow, but interactive rates (6 frames/s). The recent advances in multicore  
608 technology (Gschwind et al. 2006) and computer graphics dedicated hardware (NVIDIA 2007),  
609 and their use for scientific research (Khanna 2007; Sijbers & Batenburg 2008) give the  
610 impression that barriers to real time simulations and interactivity will soon be breached.



611 Reynolds (2006), for example, reported a multicore solution, which takes advantage of the Sony  
612 PlayStation 3 Cell processor for running simulations of 10,000 fish with animated output of  
613 cinematic quality at 60 frames/s. More recently, at the SIGGRAPH 2008 conference, the Game  
614 Computing Applications Group of AMD, Inc. was showing a cinematic quality technology demo  
615 titled 'March of the Froblins' (AMD 2008; Shopf et al. 2008), a graphics processing unit (GPU)  
616 based crowd simulation of 65,000 agents at 30 frames/s.

617 The techniques used for achieving high frame rates might at times be at the expense of  
618 biological realism. It is also true that for a scientific study centred on behaviour, the ability to  
619 interactively change the model's parameters and observing the effects in real time is a welcome  
620 plus. In computer animation for games and virtual reality (Brogan et al. 1998) high frame rates  
621 are important and the modelling of flocking behaviour has a niche of its own—it falls under the  
622 subject of 'controlling groups of objects'. Disregarding the cost of achieving the desired degree  
623 of visual realism, the simplicity of achieving high frame rates depends on the class of the  
624 controlled group of objects. Using Parent's (2002) terminology, there are three principal classes  
625 of controlled groups: 1) 'particles', characterized as large collections of individual objects, each  
626 of which obeys simple physical laws, such as momentum and conservation of energy, but has no  
627 'intelligence', or decision making capacity; usually, such particles interact mostly with their  
628 environment, and there is little, if any, inter-individual exchange (typical examples are models  
629 of fluids, gaseous phenomena, hair, fur, etc.); 2) 'flocks', characterized as medium (fewer in  
630 number than particles) size collections of individual objects, with some incorporated physics  
631 and intelligence—interaction with the environment and inter-individual exchange (typical  
632 examples are models of schools, swarms, herds, crowds, traffic, etc.); 3) 'autonomous agents',  
633 characterized as small collections, with little, if any, incorporated physics and much intelligence  
634 (typical examples are intelligent agents, autonomous robots, software agents, computer viruses,  
635 etc.). All three classes are examples of independently behaving members of groups with varying  
636 levels of autonomy, physical characteristics and simulated motions.

637 Kennedy & Eberhart (1995) were the first to incorporate elements from artificial life and  
638 artificial intelligence (AI) studies to allow bird flocking behaviour models to serve as exemplars  
639 for more general kinds of behaviour, including human social behaviour (Helbing & Molnár  
640 1995). A group of interacting animats is a 'swarm' in AI terms, and Kennedy & Eberhart (1995)  
641 presented algorithms by which a swarm might optimize its behaviour, or adapt to serve some  
642 end, such as increasing energy input. The animats in a swarm make decisions about their own  
643 behaviour based on the behaviour and knowledge gained from their neighbours, as well as the  
644 perceived elements of their surroundings, such as locations of 'feeding areas'. From the swarm's  
645 collective behaviour emerges the animats' indirect approach to relatively-good solutions. These  
646 algorithms are also known as 'particle swarm optimization', or PSO for short (Kennedy et al.  
647 2001; Engelbrecht 2006). Macgill & Openshaw (1998) and Macgill (2000), for example, later on  
648 used flocking behaviour to assist the analysis of geographical data. Subsequent AI studies  
649 started introducing more and more intelligence in individual animats while reducing their  
650 number; Odell (1998) provided a summary of the terminology and properties attributed to  
651 'agents' in computer studies.

652 In the mid-1990s, physicists began to show interest in the mathematics and physics of  
653 organized flocks, using a perspective very different from those found in earlier biological and  
654 aerodynamic studies. Vicsek et al. (1995) and Toner & Tu (1995, 1998) viewed the birds in a  
655 flock as particles, behaving much as molecules in a fluid or atoms in a crystal might, and that  
656 they were responsive to the same mathematical rules. The models were all based around the  
657 same perception model as in Heppner & Grenander (1990); inter-individual influences occurred  
658 between animats that were not further apart than a predefined distance. Additionally, an overall  
659 constant flight speed was assumed and the number of drives was reduced to merely one,  
660 attempting to match flight direction with nearby neighbours. A stochastic component had been  
661 added in these models, perhaps on similar presumptions as the random impact used by  
662 Heppner & Grenander (1990). As theoretical physicists, Toner & Tu (1998, p. 4830) may have  
663 been somewhat removed from realities in the field when they suggested, 'This correlation

664 function should be *extremely easy* (italics added) to measure in simulations, and in experiments  
665 on real herds or flocks, in which, say, video tape allows one to measure the positions ... of all the  
666 birds ... in the flock at a variety of times  $t'$ . Vicsek et al. (1995) suggested that the physics  
667 concepts associated with phase transitions, as in the transition from solid to liquid forms of  
668 materials, might serve to explain the puzzling shifts between orderly and disorderly flock  
669 formations often seen in birds like European Starlings. All in all, their models represent a  
670 substantial simplification in biological assumptions over the initial ones proposed by Okubo  
671 (1986), Reynolds (1987) and Heppner & Grenander (1990). Using Parent's (2002) terminology,  
672 these models fall perfectly under the particles category. The model devised by Vicsek et al.  
673 (1995) is also known as the 'self-propelled particles' model, or SPP. Physicists embraced these  
674 minimalist models and a number of subsequent studies have been published (see Czirók et al.  
675 1997; Czirók & Vicsek 1999, 2000; Tu 2000; Li et al. 2007; Li & Xi 2008; Chaté et al. 2008; Gönci  
676 et al. 2008; Huepe & Aldan 2008, for example). Recent field observations by Ballerini et al.  
677 (2008a, b) and Cavagna et al. (2008a, b) seem to be making an impact in the physics community  
678 and even physicists are starting to acknowledge the importance of inclusion of attractive-  
679 repulsive drives (Grégoire et al. 2003; Grégoire & Chaté 2004; Feder 2007). An additional result  
680 of these field observations, is one that somewhat contradicts the approach commonly assumed  
681 by flock flight models. These typically assume a fixed radius of interaction. Data obtained by  
682 Ballerini et al. (2008a, b) and Cavagna et al. (2008a, b) seems, on the other hand, to suggest that  
683 it is not the radius, but the number of influencing individuals, which remains constant.

684 Just as Kennedy & Eberhart (1995) incorporated AI elements into flight flock models to  
685 devise PSO, other computer science studies applied AI algorithms to evolve the models  
686 themselves. Reynolds (1993a, b), Zaera et al. (1996), and Spector et al. (2005), used genetic  
687 programming, a technique for automatically creating computer programs that satisfy a specified  
688 fitness criterion, to evolve the individual animat's rules, or in this case 'programs', which, when  
689 the animats interacted, produced flocking behaviour. All previous models employed constants  
690 like 'perception radius' (diameter of the sphere centred at the currently observed animat's

691 origin, by the use of which nearby neighbours are selected), ‘weights’ (typically the direction of  
692 flight [and flight speed] of the currently observed animat is computed as a weighted sum of the  
693 individual ‘desired’ flight directions that would meet the individual drives, respectively), etc.  
694 Heppner & Grenander’s (1990) approach was to modify these by hand and analyse the results.  
695 The AI approach was to use evolutionary computing. Genetic algorithms were used to vary the  
696 parameters to optimize the behaviour to a specified fitness criterion. Dimock & Selig (2003), for  
697 example, used genetic algorithms on a modified Reynolds (1987) model to find parameters for  
698 minimum power consumption in a flock of simulated birds. Wood & Ackland (2007), on the  
699 other hand, using a Couzin et al. (2002) model, studied the evolution of group formation when  
700 subjected to simulated predation and foraging. Their results replicate conventional evolutionary  
701 behaviour—foraging animats prefer a narrower perception volume, while the hunted prefer a  
702 wider one.

703 Couzin et al. (2002) and Couzin & Krause (2003) added the next level of sophistication in  
704 flocking models. The substantial difference was not in the animats’ drives, but in the perception  
705 model, or when these drives were actually in effect. Whereas Heppner & Grenander (1990) used  
706 one perception volume for all three drives, Reynolds (1987, 1999, 2004) three non-exclusive  
707 perception volumes with biologically inspired limitations, Couzin et al. (2002) and Couzin &  
708 Krause (2003) introduced a different approach; in their model there were three exclusive  
709 perception volumes, or ‘zones’, using their terminology: 1) ‘zone of repulsion’, 2) ‘zone of  
710 orientation’ and 3) ‘zone of attraction’. If there were neighbours in the zone of repulsion, then  
711 only the separation drive was active and the other two ignored. If, however, there were no  
712 neighbours in it, the other two drives were averaged, but the animat attempted to match  
713 velocity only with the neighbours in the zone of orientation and attempted to stay close only to  
714 the neighbours in the zone of attraction. Additionally, the zone of repulsion was modelled as a  
715 sphere, whereas the other two were modelled as a sphere with a blind cone subtracted at the  
716 animats’ back. Couzin et al. (2002) considered what would happen to group movements if  
717 individuals in the group modified their behavioural rules in response to experience with the

718 flock as a whole. More specifically, what would happen if the diameter of the zone of orientation  
719 was variable, while keeping the zones of repulsion and attraction constant. They found that, as  
720 the diameter of the zone of orientation increased, the group went from a loosely packed  
721 stationary swarm, to a torus where individuals circle round their centre of mass and, finally, to a  
722 parallel group moving in a common direction (see also Sumpter 2006). Further on they  
723 discovered that the transitions were rapid, and as the diameter decreased, the collective  
724 behaviour was different. They established that two completely different behavioural states can  
725 exist for identical parameters, and that transition between behavioural states depends on the  
726 previous history (structure) of the group, even though the individuals have no explicit  
727 knowledge of what that history is. Consequently, they suggested that the system exhibits a form  
728 of 'collective memory'. In a later study, Couzin et al. (2005) examined leadership and decision  
729 making in animal groups on the move by giving knowledge of a preferred flight direction only to  
730 a proportion of the simulated animals. The study revealed that the larger the group, the smaller  
731 the proportion of informed individuals needed to guide the group, and that only a small  
732 proportion is required to achieve great accuracy. Several recent experimental studies (Biro et al.  
733 2006; Codling et al. 2007; Dell'Arciccia et al. 2008) investigated the 'many-wrongs principle' in  
734 pigeon homing and suggest that pigeons flying in a group have better navigational performance  
735 than birds flying alone, but it is not clear whether the spatial organization of the flock is  
736 significant in this observation.

737 Lebar Bajec et al. (2003a, b, 2005) and Lebar Bajec (2005) introduced the concept of fuzzy  
738 logic to flocking models. The basic concept of the model remained the same; three drives and  
739 perception modelled as a sphere with a blind cone removed from the back. But in previous  
740 models, the animats would react to their surroundings in a 'crisp' way. For example, if we are  
741 interested in two moving animats that are on a closing course with one another, there might be  
742 some specific threshold distance at which they would deviate to avoid collision (e.g. when they  
743 enter each other's zone of repulsion [Couzin et al. 2002]). Or, in a slightly more complex  
744 example, there might be a gradient for different closing angles such that the animats would

745 deviate proportionately, but still in deterministic fashion depending on the closing angle.  
746 However, with fuzzy logic, vague qualities like 'close' or 'far' rather than a specific distance or  
747 angle can be used to describe the behavioural repertoire of the animat. In this fashion, a more  
748 naturalistic type of behaviour can be produced. Indeed, Heppner & Grenander (1990) used a  
749 single perception volume, Reynolds (1987, 1999, 2004) three overlapping perception volumes,  
750 and Couzin et al. (2002) and Couzin & Krause (2003) advanced the model by introducing three  
751 non-overlapping perception zones, the use of vague qualities enabled Lebar Bajec et al. (2005)  
752 to produce a mixture of these approaches with partially overlapping perception zones. The issue  
753 this model has, with respect to the others, is that it is two dimensional; animats can move left or  
754 right, but not up or down. As real birds exist in three dimensions, a genuinely realistic  
755 simulation needs to feature the third dimension. Moškon et al. (2007) expanded the fuzzy model  
756 to account for foraging behaviour by including hunger as a drive. While doing so, they also  
757 modelled foraging fields and landing and taking off from them; while this has not been achieved  
758 by promoting the drives to work in three dimensions, they upgraded the model to pseudo 3D  
759 nonetheless.

## 760 So, How do Birds Seem to Turn and Wheel Together?

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761 In the 1970s, there was no conceptual alternative to a leadership model for producing  
762 simultaneous or near-simultaneous turning movements in cluster flocks. With the advent of the  
763 many models that treat flocks as collections of independently acting agents that produce turning  
764 movements as the product of individual movement decisions, a viable alternative to leadership  
765 models now exists, but such models 1) do not rule out the possibility that under certain  
766 circumstances, particularly with small, or family flocks, leadership might still play a role in  
767 cluster flock movements, and 2) do not provide evidence that birds use the same algorithms as  
768 the models. Just as there may be several biological functions for line formations, it may be that  
769 there are multiple mechanisms for producing cluster flock movements.

## 770 **CONCLUSION**

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771 Advances in the understanding of the function and mechanisms of organized flight have been  
772 strongly linked to the introduction of new techniques or technologies. Heppner (1997)  
773 identified several areas that might be expected to produce such advances, but a decade later,  
774 although it has been possible to refine and more closely define these needs, much still needs to  
775 be done.

- 776 1. Three dimensional simulations. Some of the existing simulations (Vicsek et al. 1995;  
777 Lebar Bajec et al. 2005; Mořkon et al. 2007; Nathan & Barbosa 2008), although  
778 capable of producing realistic-appearing flocks on a computer screen, feature animats  
779 that travel in a two dimensional universe. They may travel left or right, but not up or  
780 down. As real birds exist in a 3D world, a genuinely realistic simulation would have to  
781 feature the third dimension. Adding the additional dimension is not a trivial  
782 programming task, but its accomplishment could be expected to pay large dividends.
- 783 2. Non-homogeneous models. To date, flight flock models have assumed that flocks are  
784 composed of identical subjects. In reality, there will be individual differences in age,  
785 gender, sensitivity to hunger, health, and other factors that may well influence the  
786 collective behaviour of the flock.
- 787 3. Fast, cheap, field data acquisition. Cavagna et al.'s (2008a) technique for obtaining the  
788 3D positions of thousands of birds in a flock has yielded remarkable results, but the  
789 method requires custom-made synchronizing equipment for the cameras, skilled  
790 operators, lengthy processing, and a fixed location. As a result, it is difficult to  
791 compare species, conditions, or fine structure over time. The current generation of  
792 digital still and video cameras offers the potential for both high resolution and a high  
793 frame rate at a reasonable cost. Commercial wireless technology, such as that used to  
794 simultaneously fire multiple remote flash units, offers the potential of synchronizing

795 two (or more) cameras in the field without the necessity for custom made  
796 synchronizing devices.

797 4. User-friendly simulations. The current generation of flocking simulations is primarily  
798 designed to be used and manipulated by their designers, who may or may not be  
799 familiar with the behaviour of animals in the field. The programs are not easily used  
800 or modified by other users unfamiliar with programming. It would be very helpful if  
801 future simulations came with a 'console', or control panel that would allow non-  
802 programmers to change the parameters or their values in the simulation, such as  
803 preferred velocity, or attractiveness of feeding site, thus allowing field biologists to  
804 examine the results of changing inputs to the program based on their field experience.  
805 It might also be possible to set up 'detectors' in the program, as is done in  
806 experimental particle physics, to allow many different combinations of parameters  
807 and values to be run in sequence, and the program would flag interesting behaviours,  
808 such as the appearance of a V, when they appear. For example, the Boston Museum of  
809 Science in Massachusetts has a large public display called the 'Virtual Fishtank'  
810 (Nearlife, Inc. 2001) that enables visitors to interactively change the behaviour of  
811 individual fish in a 'school', and immediately see the change in the behaviour of the  
812 school.

813 5. Metrics for 'truth testing'. Current simulations offer naturalistic appearing virtual  
814 flocks, but it cannot be certain that real birds use the same algorithms employed in  
815 the simulation. Ideally, one would produce a simulation of a particular species'  
816 flocking behaviour, and use it to make predictions about the behaviour of the real  
817 flock, and then test those predictions in the field. To do this, one would have to have a  
818 metric that could be derived from the simulation, and then measured in the field. For  
819 example, some simulations produce flocks that apparently turn and wheel much like  
820 real flocks. Perhaps 'turning and wheeling' could be quantified, such that one could  
821 say that, for example, a flock of X number of birds of species Y will make a turn,



822 defined as a departure of more than 20° from the mean direction exhibited in the  
823 previous 5 s, every 8.2 s. If this variable were measurable in the field, it could then be  
824 possible to refine the model to produce more accurate predictions. Successful  
825 prediction would, of course, not be prima facie evidence that the algorithms in the real  
826 and virtual worlds were the same, but would certainly provide stronger evidence than  
827 a superficial, qualitative similarity. Dill et al. (1997) discussed this issue more  
828 extensively.

829 The last 40 years have seen remarkable progress in the understanding of this intriguing and  
830 aesthetically spectacular phenomenon. In addition to being a phenomenon worthy of  
831 examination in its own right, the study of organized flight in birds has provided a model system  
832 that has demonstrated utility in the study of crowd behaviour, bird strikes on aircraft, traffic  
833 theory, complex systems, particle swarms, computer animation, and control of (remotely  
834 piloted) autonomous aircraft. At this time, it is possible to foresee that with the assistance of  
835 biologists, physicists, mathematicians and computer scientists working together, we will, before  
836 long, truly be able to say how and why birds fly in organized groups.

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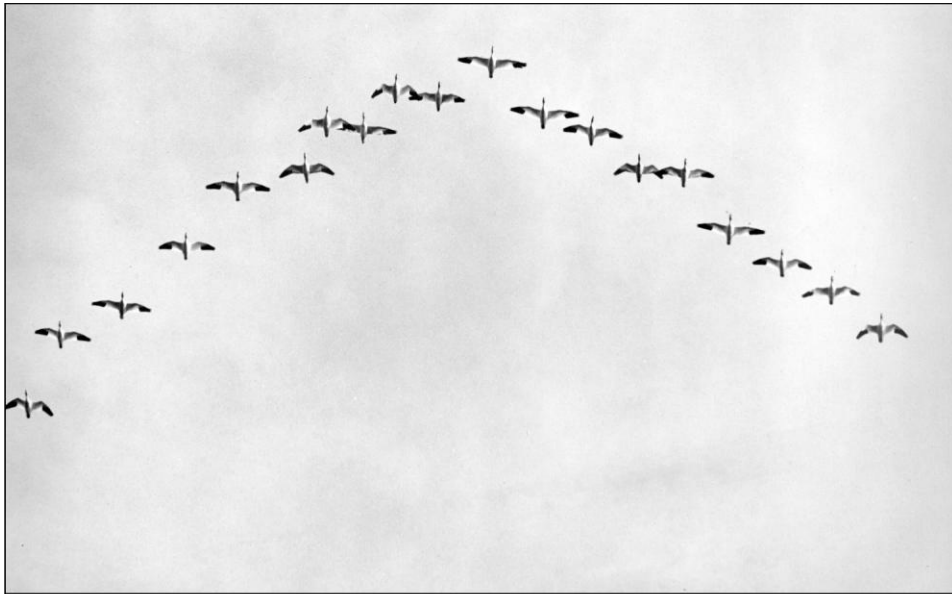
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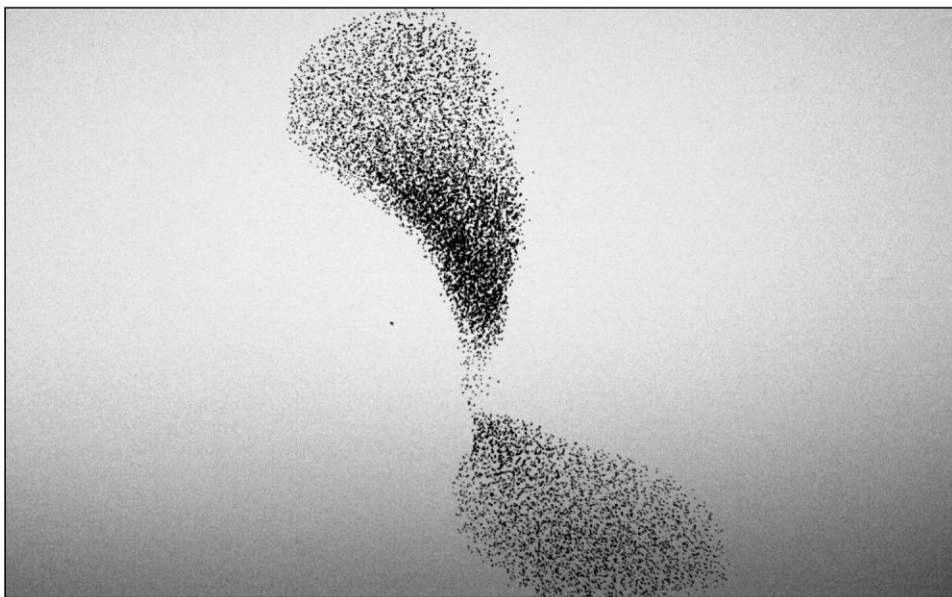
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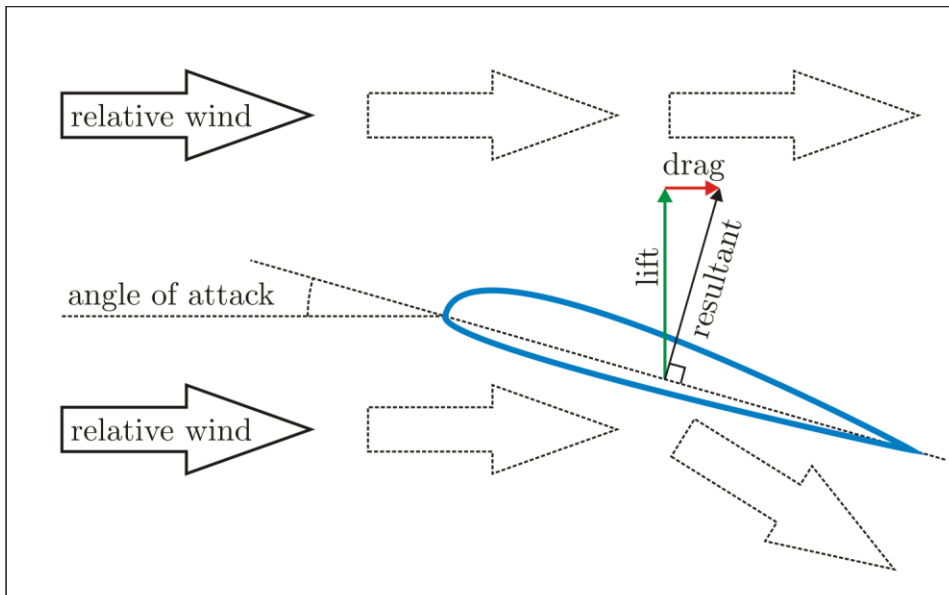
A) Line formation



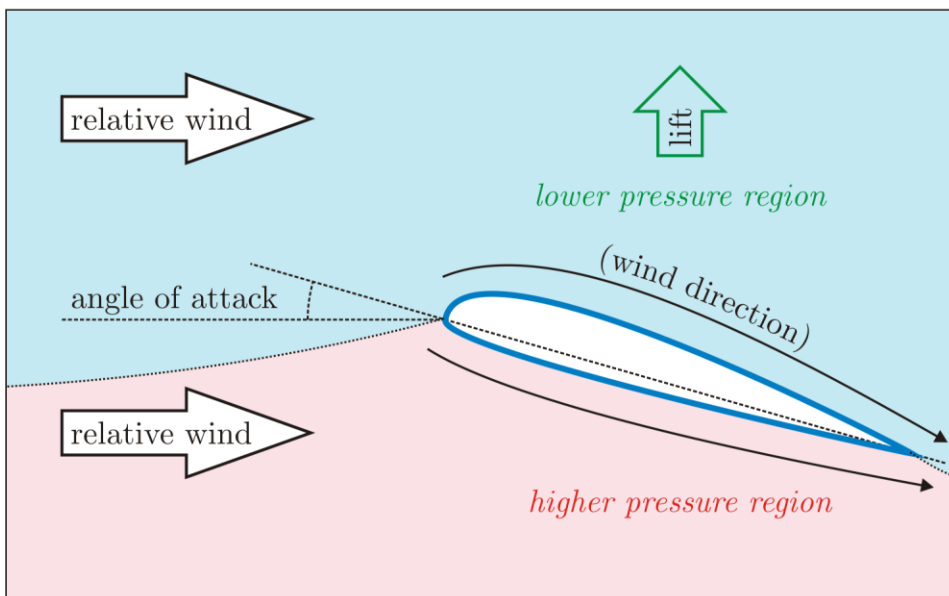
B) Cluster formation

1154 **Figure 1.** A) Line formation of Snow Geese, *Chen hyperborea*. B) Cluster formation of European  
1155 Starlings, *Sternus vulgaris*, over Rome (© 2009 STARFLAG project, INFN-CNR).



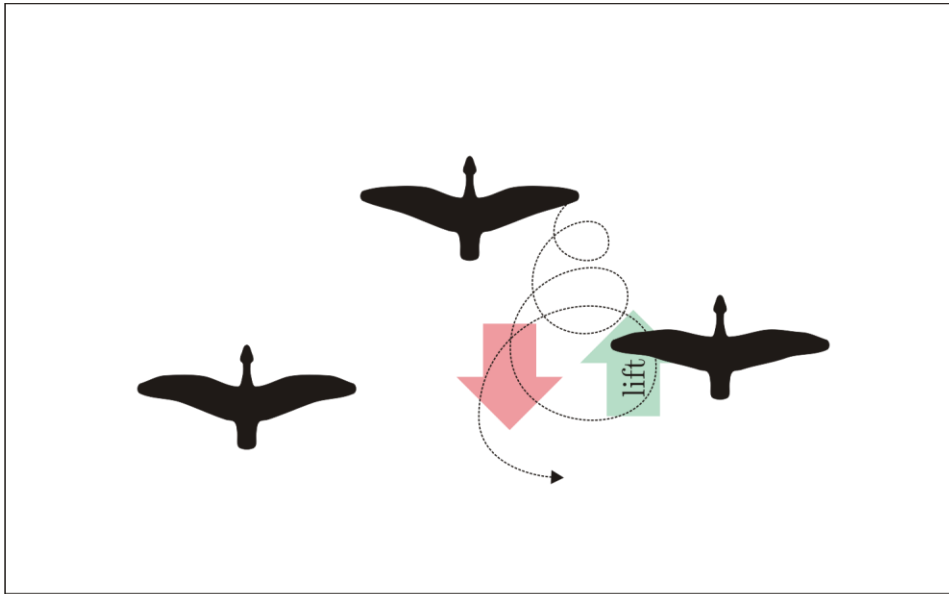


A) Newtonian lift

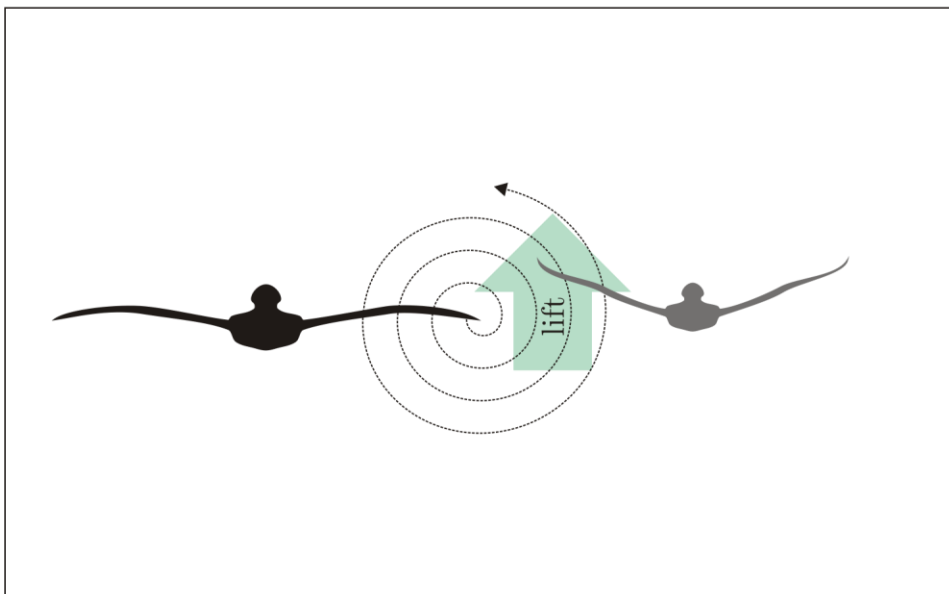


B) Bernoulli lift

1156 **Figure 2.** Lift is generated in two principal ways. A) Air is deflected downward when there is a  
 1157 positive angle of attack between a wing and the relative wind. This deflection produces a  
 1158 reactive 'Newtonian lift' force. B) When air moves faster over the top of the wing than the  
 1159 bottom, a reduced pressure area is created on the top of the wing, generating 'Bernoulli lift'.



A) Top view



B) Front view

1160 **Figure 3.** Air streams off of a bird's wingtip as a horizontal vortex that has a rising and falling  
 1161 component. If the wingtip of a following bird were positioned in the rising component of the  
 1162 vortex generated by a preceding bird, some of the energy lost by that bird into the tip vortex, as  
 1163 a product of generating lift, might be recaptured by a following bird.